1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$
2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$
3. Find the equations of the tangent and normal to curve $x^2$ (square) $+ y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

Soln

(1)

$$y - 3x - 2 = 0$$
$$3y + x + 9 = 0$$

For the lines to be perpendicular then $m_1 m_2 = -1$

$$y - 3x - 2 = 0$$

making y the subject of the formula

$$y = 3x + 2$$
$$y = 3x + 2$$

By comparison with $y = mx + c$

$$m_1 = 3$$

$$3y + x + 9 = 0$$

making y the subject of the formula

$$3y = -x - 9$$
$$y = \frac{-x}{3} - \frac{9}{3}$$
$$y = -\frac{1}{3} - 3$$

$y = mx + c$ $\quad m_2 = -\frac{1}{3}$

$m_1 m_2 = -1$ $\quad$ For perpendicularity

$3 \times \frac{1}{3} = -1$ $\quad$ Since $m_1 m_2 = -1$, lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

2 $3y - 4 = 2x + 3$ — ①

$y - 5 = x + 6$ — ②

Making y the subject of formula in 1

$$3y = 2x + 3 + 4$$
$$3y = 2x + 7$$
$$y = \frac{2x}{3} + \frac{7}{3}$$

By comparing with $y = mx + c$

$$m_1 = \frac{2}{3}$$

making y the subject of formula in 2

$$y - 5 = x + 6$$
$$y = x + 6 + 5$$
$$y = x + 11$$

By comparing with $y = mx + c$

$$m_2 = 1$$

For perpendicularly, $m_1 m_2 = -1$

$m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$, thus not equal.

Hence the lines $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular

3. $x^2 + y^2 + 3xy - 11 = 0$

$$m = \frac{dy}{dx}$$

$$x^2 + y^2 + 3xy - 11 = 0$$

$$\frac{dy}{dx}$$

$$2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y \cdot 1\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3\left(2\frac{dy}{dx} + y\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \left.\frac{dy}{dx}\right|_{x=1,\, y=2} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2-6}{4+3} = \frac{-3}{7}$$

ⓐ Equation of a tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$7y + 8x - 22 = 0$ is the equation of tangent

b) Equation of normal

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 2 = \frac{-1}{-8/7}(x - 1)$$

$$y - 2 = 7/8\,(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x \cancel{-} + 7 \cancel{+} 16 = 0$$

$$8y - 7x + 7 - 16 = 0$$

$8y - 7x - 9 = 0$ is the equation of the normal